$B \to K_0^*(1430)K$ decays in the perturbative QCD approach *

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Abstract

In this article, we calculate the branching ratios of $B \to K_0^*(1430)K$ decays by employing the perturbative QCD (pQCD) approach at leading order. We perform the evaluations in the two scenarios for the scalar meson spectrum. We find that (a) the leading order pQCD predictions for the branching ratio $Br(B^+ \to K^+\bar{K}_0^*(1430)^0)$ which is in good agreement with the experimental upper limit in both scenarios, while the pQCD predictions for other considered $B \to K_0^*(1430)K$ decay modes are also presented and will be tested by the LHC experiments; (b) the annihilation contributions play an important role in these considered decays, for $B^0 \to K_0^*(1430)^{\pm}K^{\mp}$ decays, for example, which are found to be $(1-4)\times 10^{-6}$.

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It is well-known that the scalar meson spectrum is one of the interesting topics for both experimental and theoretical studies, but the underlying structure of the light scalar mesons is still controversial. Perhaps, the $B \to SP$ decays can give us the opportunity to receive new understanding on the scalar meson.

On the theory side, up to now, some two body non-leptonic B meson involving a scalar $K_0^*(1430)$(For the sake of simplicity, we will use K_0^* to denote $K_0^*(1430)$ in the following section) meson decays have been studied by using various theoretical methods or approaches, for example, in Ref. [1–4], where the authors investigated the properties of K_0^* by calculating the branching ratios, CP-violating asymmetries and other physical quantities. In this paper, based on the assumption of two-quark structure of scalar K_0^* meson, we will calculate the branching ratios of $B^+ \to K_0^{*+}\bar{K}^0, K^+\bar{K}_0^{*0}$ and $B^0/\bar{B}^0 \to K_0^{*0}\bar{K}^0, K^0\bar{K}_0^{*0}, K_0^{*+}K^-, K^+K_0^{*-}$ decays directly by employing the low energy effective Hamiltonian [5] and the pQCD factorization approach [6–9].

On the experimental side, only one upper limit on $B^+ \to \bar{K}_0^{*0}K^+$ is available now [10, 11] (upper limits at 90% C.L.):

$$Br(B^+ \to \bar{K}_0^{*0}K^+) \ < \ 2.2 \times 10^{-6} \ . \tag{1}$$

But this situation will be improved rapidly when the LHC experiment starts to run in 2009.

This paper is organized as follows. In Sec. I, we calculate analytically the related Feynman diagrams and find the various decay amplitudes for the studied decay modes. In Sec. II, we show the numerical results for the branching ratios of $B \to K_0^*K$ decays. A short summary and some discussions are also included in this section.

I. PERTURBATIVE CALCULATIONS OF $B \to K_0^*K$ DECAYS

In the pQCD factorization approach, the decay amplitude of $B \to K_0^*K$ decays can be written conceptually as the convolution,

$$\mathcal{A}(B \to K_0^*K) \ \sim \ \int dx_1 dx_2 dx_3 b_1 db_1 b_2 db_2 b_3 db_3 \times \mathrm{Tr}\left[C(t)\Phi_B(x_1,b_1)\Phi_{K_0^*}(x_2,b_2)\Phi_K(x_3,b_3)H(x_i,b_i,t)S_t(x_i)\, e^{-S(t)}\right], \tag{2}$$

where the term "Tr" denotes the trace over Dirac and color indices. $C(t)$ is the Wilson coefficient. The function $H(x_i, b_i, t)$ is the hard part and can be calculated perturbatively, while b_i is the conjugate space coordinate of k_{iT}, and t is the largest energy scale in hard function. The function Φ_M is the wave function which describes hadronization of the quark and anti-quark to the meson M. The threshold function $S_t(x_i)$ smears the end-point singularities on x_i. The last term, $e^{-S(t)}$, is the Sudakov form factor which suppresses the soft dynamics effectively.

The low energy effective Hamiltonian for decay modes $B \to K_0^*K$ can be written as

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}}\left[V_{ub}^*V_{ud}\left(C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu)\right) - V_{tb}^*V_{td}\sum_{i=3}^{10} C_i(\mu)\, O_i(\mu)\right] \ , \tag{3}$$

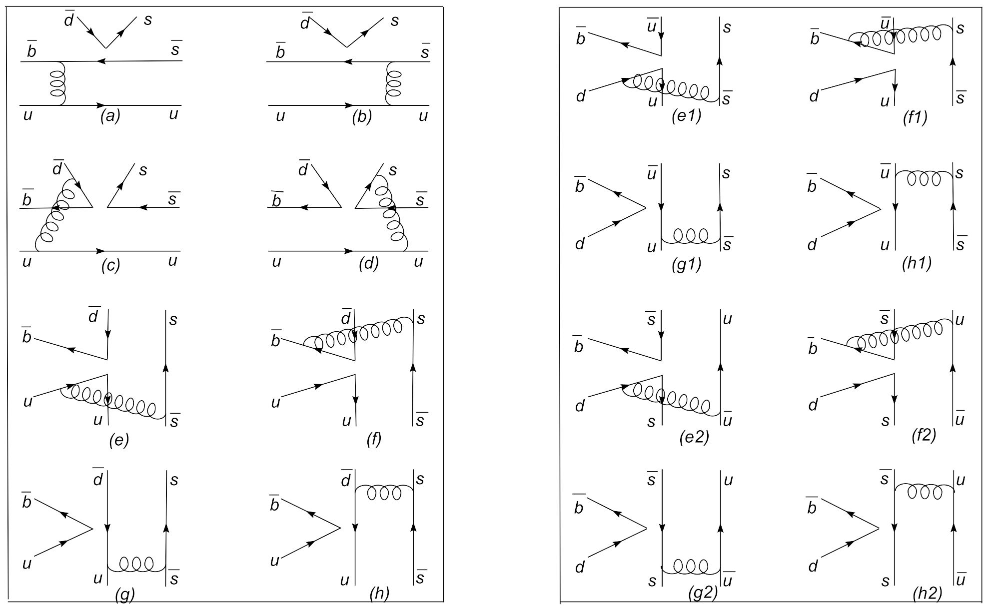

FIG. 1: Typical Feynman diagrams contributing to $B^+ \to K_0^{*+}\bar{K}^0(B^+ \to K^+\bar{K}_0^{*0})$(a-h in l.h.s.) and pure annihilation $B^0 \to K_0^{*+}K^-(B^0 \to K^+K_0^{*-})$(e1-h2 in r.h.s.) decays, respectively.

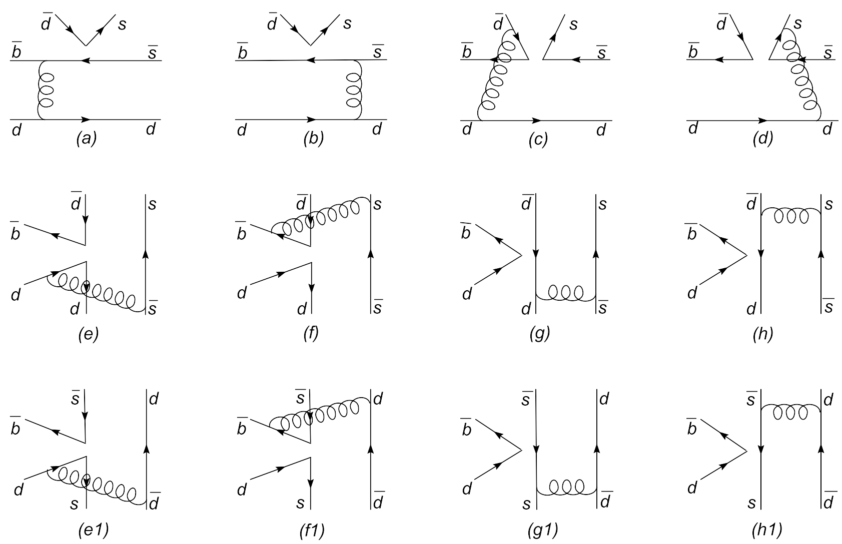

FIG. 2: Typical Feynman diagrams contributing to $B^0 \to K_0^{*0}\bar{K}^0(B^0 \to \bar{K}_0^{*0}K^0)$ decays.

where the Fermi constant $G_F = 1.16639 \times 10^{-5} GeV^{-2}$, V_{ij} is the Cabbibo-Kobayashi-Maskawa (CKM) matrix elements, $C_i(\mu)$ are the Wilson coefficients at the renormalization scale μ and O_i are the four-fermion operators for the case of $\bar{b} \to \bar{d}$ transition.

The B meson is treated as a heavy-light system. We here use the same B meson wave function as in Ref. [12–14], while the treatment for the scalar meson K_0^* is that same as in Ref. [4]. For the distribution amplitudes of light pseudoscalar K meson, we directly adopt the form as given in Ref. [15].

At leading order, the relevant Feynman diagrams for the $B^+ \to K_0^{*+}\bar{K}^0, K^+\bar{K}_0^{*0}$, $B^0 \to K_0^{*+}K^-, K^+K_0^{*-}$ and $B^0 \to K_0^{*0}\bar{K}^0, K^0\bar{K}_0^{*0}$ decays have been shown in Figs. 1

and 2. Note that, on the other hand, $\bar{B}^0$ meson can also decay into the same final states $K_0^{*+}K^-, K^+K_0^{*-}$ and $K_0^{*0}\bar{K}^0, K^0\bar{K}_0^{*0}$ simultaneously.

Based on the assumption of two quark structure of scalar K_0^* meson, by analytical calculations of the relevant Feynman diagrams and combining the contributions from different diagrams, one can find the total decay amplitudes for the considered decays:

$$
\begin{aligned}
\mathcal{M}(B^+ \to K_0^{*+}\bar{K}^0) &= -\xi_t \Big\{ f_K F_{eK_0^*}(a_4 - a_{10}/2) + f_K F^{P2}_{eK_0^*}(a_6 - a_8/2) + M_{eK_0^*}(C_3 - C_9/2) \\
&\quad + M^{P1}_{eK_0^*}(C_5 - C_7/2) + F^{P2}_{aK_0^*}(a_6 + a_8) + M^{P1}_{aK_0^*}(C_5 + C_7)\Big\} \\
&\quad + M_{aK_0^*}\left\{\xi_u C_1 - \xi_t(C_3 + C_9)\right\} + F_{aK_0^*}\left\{\xi_u a_1 - \xi_t(a_4 + a_{10})\right\} \qquad (4)\\
\mathcal{M}(B^0 \to K_0^{*0}\bar{K}^0) &= -\Big\{(a_4 - a_{10}/2)\xi_t f_K F_{eK_0^*} + (a_6 - a_8/2)(\xi_t f_K F^{P2}_{eK_0^*} + \xi_t F^{P2}_{aK_0^*}) + (C_3 \\
&\quad - C_9/2)\xi_t M_{eK_0^*} + (C_5 - C_7/2)(\xi_t M^{P1}_{eK_0^*} + \xi_t M^{P1}_{aK_0^*}) + (C_3 + C_4 \\
&\quad - (C_9 + C_{10})/2)\, \xi_t M_{aK_0^*} + (C_6 - C_8/2)(\xi_t M^{P2}_{aK_0^*} + \xi_t M^{P2}_{a\bar{K}}) \\
&\quad + (a_3 + a_4 - a_5 + (a_7 - a_9 - a_{10})/2)\, \xi_t F_{aK_0^*} + (C_4 - C_{10}/2)\xi_t \\
&\quad \cdot M_{a\bar{K}} + \xi_t F_{a\bar{K}}\,(a_3 - a_5 + (a_7 - a_9)/2)\Big\}\ , \qquad (5)\\
\mathcal{M}(B^0 \to K_0^{*+}K^-) &= M_{aK_0^*}\left\{\xi_u C_2 - \xi_t(C_4 + C_{10})\right\} - (C_6 + C_8)\xi_t M^{P2}_{aK_0^*} - (C_4 \\
&\quad - C_{10}/2)\xi_t M_{aK} + F_{aK_0^*}\left\{\xi_u a_2 - \xi_t(a_3 - a_5 - a_7 + a_9)\right\} \\
&\quad - (C_6 - C_8/2)\xi_t M^{P2}_{aK} - (a_3 - a_5 + (a_7 - a_9)/2)\, \xi_t F_{aK} \qquad (6)
\end{aligned}
$$

where $\xi_u = V_{ub}^* V_{ud}$, $\xi_t = V_{tb}^* V_{td}$. The individual decay amplitudes for $B \to K_0^* K$ decays, such as $F_{eK_0^*}$ and $F^{P2}_{eK_0^*}$, etc, are similar to those for $B \to K_0^*\eta^{(\prime)}$ decays as given in Ref. [4], and can be obtained easily by the replacement of $\eta^{(\prime)} \to K$.

The Wilson coefficients a_i in Eq. (4-6) are the combinations of the ordinary Wilson coefficients $C_i(\mu)$,

$$
a_1 = C_2 + C_1/3, \quad a_2 = C_1 + C_2/3, \quad a_i = C_i + C_{i\pm 1}/3, \quad i = 3 - 10. \qquad (7)
$$

where the upper (lower) sign applies, when i is odd (even).

The expressions of total decay amplitudes for $B^+ \to \bar{K}_0^{*0}K^+$ and $B^0 \to \bar{K}_0^{*0}K^0, K^+K_0^{*-}$ modes can be easily obtained with the replacement of $K_0^* \to K, \bar{K} \to \bar{K}_0^*$[here, $K_0^*(K)$ and $\bar{K}_0^*(\bar{K})$ denote $K_0^{*+,0}(K^{+,0})$ and $K_0^{*-}, \bar{K}_0^{*0}(K^-, \bar{K}^0)$] in Eq. (4,5,6), respectively.

II. NUMERICAL RESULTS AND DISCUSSIONS

For numerical calculation, we will use the following input parameters:

$$
\begin{aligned}
\Lambda^{(f=4)}_{\overline{\mathrm{MS}}} &= 0.250\mathrm{GeV}, \quad f_K = 0.16\mathrm{GeV}, \quad f_B = 0.190\mathrm{GeV}, \\
m_0^K &= 1.6\mathrm{GeV}, \quad m_{K_0^*} = 1.425\mathrm{GeV}, \quad M_W = 80.41\mathrm{GeV}, \\
M_B &= 5.28\mathrm{GeV}, \quad \tau_{B^\pm} = 1.638\times 10^{-12}\mathrm{s}, \quad \tau_{B^0} = 1.53\times 10^{-12}\mathrm{s}. \qquad (8)
\end{aligned}
$$

For the CKM matrix elements, here we adopt the Wolfenstein parametrization for the CKM matrix, and take $\lambda = 0.2257, A = 0.814, \bar{\rho} = 0.135$ and $\bar{\eta} = 0.349$ [10].

In the two-quark picture of the scalar meson K_0^*, there are two scenarios for the choice of the decay constants $f_{K_0^*}$, $\bar{f}_{K_0^*}$ and the Gegenbauer moments B_1 and B_3 [1]:

$$\begin{aligned} f_{K_0^*} &= -0.025 \pm 0.002\text{GeV}, \quad \bar{f}_{K_0^*} = -0.300 \pm 0.030\text{Gev}, \\ B_1 &= 0.58 \pm 0.07, \qquad\qquad\quad B_3 = -1.20 \pm 0.08, \end{aligned} \tag{9}$$

in Scenario I, and

$$\begin{aligned} f_{K_0^*} &= 0.037 \pm 0.004\text{GeV}, \quad \bar{f}_{K_0^*} = 0.445 \pm 0.050\text{Gev}, \\ B_1 &= -0.57 \pm 0.13, \qquad\qquad B_3 = -0.42 \pm 0.22, \end{aligned} \tag{10}$$

in Scenario II [1]. In the numerical calculations we will consider these two scenarios, respectively.

TABLE I: The leading order pQCD predictions for the branching ratios (in unit of 10^{-6}) of $B \to K_0^* K$ decays in both scenarios, where the numbers in parentheses are the central values of branching ratios without the inclusion of annihilation diagrams. For comparison, we also cite the experimental upper limit as given in Ref. [10, 11].

Modes	Scenario I	Scenario II	Data
$B^+ \to K_0^{*+} \bar{K}^0$	$1.5^{+0.7+0.3+0.1+0.4}_{-0.4-0.2-0.1-0.2}(2.3)$	$5.0^{+1.8+0.4+0.9+1.2}_{-1.2-0.3-0.6-1.0}(5.1)$	–
$B^+ \to \bar{K_0^*}^0 K^+$	$1.2^{+0.2+0.1+0.1+0.2}_{-0.1-0.1-0.1-0.2}(0.8)$	$2.2^{+0.6+0.2+0.4+0.5}_{-0.4-0.2-0.1-0.4}(1.8)$	< 2.2
$B^0/\bar{B}^0 \to K_0^{*0} \bar{K}^0$	$2.7^{+0.5+0.5+0.4+0.6}_{-0.3-0.4-0.4-0.5}(2.7)$	$7.5^{+2.1+0.5+1.7+1.7}_{-1.5-0.6-1.2-1.7}(6.0)$	–
$B^0/\bar{B}^0 \to \bar{K_0^*}^0 K^0$	$2.8^{+0.2+0.4+0.5+0.6}_{-0.1-0.4-0.4-0.5}(1.2)$	$5.0^{+0.8+0.7+1.8+1.2}_{-0.6-0.6-1.0-1.0}(2.7)$	–
$B^0 \to K_0^{*0} \bar{K}^0 + \bar{K_0^*}^0 K^0$	$5.1^{+1.0+0.8+0.8+1.1}_{-0.6-0.7-0.8-1.0}(5.3)$	$14.9^{+4.3+1.3+2.7+3.5}_{-2.9-1.3-1.8-3.2}(12.0)$	–
$B^0/\bar{B}^0 \to K_0^{*-} K^+$	$3.7^{+0.4+0.5+0.5+0.8}_{-0.3-0.4-0.4-0.7}(0.0)$	$1.8^{+0.3+0.4+1.5+0.5}_{-0.2-0.3-0.9-0.4}(0.0)$	–
$B^0/\bar{B}^0 \to K_0^{*+} K^-$	$1.1^{+0.2+0.5+0.1+0.2}_{-0.2-0.4-0.2-0.2}(0.0)$	$1.6^{+0.4+0.4+0.8+0.4}_{-0.2-0.5-0.5-0.3}(0.0)$	–
$B^0 \to K_0^{*+} K^- + K_0^{*-} K^+$	$2.4^{+0.2+0.1+0.4+0.6}_{-0.1-0.1-0.4-0.4}(0.0)$	$1.2^{+0.2+0.1+1.2+0.3}_{-0.1-0.1-0.6-0.2}(0.0)$	–

Using the decay amplitudes obtained in last section, it is straightforward to calculate the branching ratios for $B \to K_0^* K$ decays. From the leading order pQCD predictions for these considered decays as displayed in Table I, some phenomenological discussions are in order:

(1) It is worth stressing that the theoretical predictions in the pQCD approach have relatively large theoretical errors induced by the still large uncertainties of many input parameters. As shown in Table I, in our pQCD predictions, the first error arises from the B meson wave function shape parameter $\omega_b = 0.40 \pm 0.04$. The second error is induced by the combination of the uncertainties of Gegenbauer moments $a_1^K = 0.17 \pm 0.17$ and/or $a_2^K = 0.115 \pm 0.115$. The last two errors come from the combinations of the Gegenbauer coefficients B_1 and/or B_3, and the decay constants $f_{K_0^*}$ and/or $\bar{f}_{K_0^*}$ of the scalar meson K_0^*, respectively.

(2) For $B^+ \to K^+ \bar{K_0^*}^0$ mode, one can find the the pQCD prediction for the CP-averaged branching ratio agrees with the currently available experimental upper limit in both scenarios.

(3) For the charged $B^+ \to K_0^{*+}\bar{K}^0$ and $B^+ \to K^+\bar{K}_0^{*0}$ channels, the CP-averaged branching ratios show us the different features in two scenarios: the values are approximately equal to each other for these two decays in Scenario I, while the former is twice larger than the latter in Scenario II. We also show the central values of the branching ratios with neglecting the annihilation contributions as given in Table I, one can see the difference between these considered two modes: the annihilated diagrams are destructive to $B^+ \to K_0^{*+}\bar{K}^0$ but constructive to $B^+ \to K^+\bar{K}_0^{*0}$ decays. Additionally, the annihilation contributions play a more important role in scenario I than that in Scenario II.

(4) It is a little complicate for us to calculate the branch ratios of $B^0/\bar{B}^0 \to f(= K_0^{*0}\bar{K}^0, K_0^{*+}K^-)(\bar{f}[= K^0\bar{K}_0^{*0}, K^+K_0^{*-}])$, since both B^0 and $\bar{B}^0$ can decay into the same final state f and $\bar{f}$ simultaneously. Because of $B^0 - \bar{B}^0$ mixing, it is very difficult to distinguish B^0 from $\bar{B}^0$. But it is easy to identify the final states. We therefore sum up $B^0/\bar{B}^0 \to K_0^{*0}\bar{K}^0$ as one channel, and $B^0/\bar{B}^0 \to K^0\bar{K}_0^{*0}$ as another, although the summed up channels are not charge conjugate states [16]. Similarly, we have $B^0/\bar{B}^0 \to K_0^{*+}K^-$ as one channel, and $B^0/\bar{B}^0 \to K^+K_0^{*-}$ as another. We also define the average branching ratio of the two channels following the same convention as experimental measure [10, 11]: $B^0 \to K_0^{*0}\bar{K}^0 + K^0\bar{K}_0^{*0}$ and $B^0 \to K_0^{*+}K^- + K^+K_0^{*-}$. The branching ratios for $B^0/\bar{B}^0 \to K_0^{*0}\bar{K}^0, K^0\bar{K}_0^{*0}$, $B^0/\bar{B}^0 \to K_0^{*+}K^-, K^+K_0^{*-}$, $B^0 \to K_0^{*0}\bar{K}^0 + K^0\bar{K}_0^{*0}$ and $B^0 \to K_0^{*+}K^- + K^+K_0^{*-}$ decays have already been presented in Table I.

(5) The branching ratios for $B^0/\bar{B}^0 \to K_0^{*0}\bar{K}^0(K^0\bar{K}_0^{*0})$ in Scenario II are larger than those in Scenario I. As for the annihilation corrections, one can see that they are constructive to $B^0/\bar{B}^0 \to K_0^{*0}\bar{K}^0$ nearly 0-20% and $B^0/\bar{B}^0 \to K^0\bar{K}_0^{*0}$ around 50%, respectively. By comparison, we find that the annihilation amplitudes are important in both scenarios for $B^0/\bar{B}^0 \to K^0\bar{K}_0^{*0}$ decay while more important in Scenario II than that in Scenario I for $B^0/\bar{B}^0 \to K_0^{*0}\bar{K}^0$ decay. For the branching ratio of $B^0 \to K_0^{*0}\bar{K}^0 + K^0\bar{K}_0^{*0}$, we find that the value in Scenario II is nearly three times as large as that in Scenario I, however, the annihilation contributions are destructive to the branching ratio in Scenario I while constructive to it in Scenario II and play a more important role in Scenario II than that in Scenario I.

(6) From the pQCD predictions for the pure annihilation contributions $B^0/\bar{B}^0 \to K_0^{*+}K^-(K^+K_0^{*-})$ and $B^0 \to K_0^{*+}K^- + K^+K_0^{*-}$ as shown in last three lines of Table I, we find that the leading order pQCD branching ratios from this part can amount to $(1-4) \times 10^{-6}$, which indicate the large annihilation effects in $B \to K_0^*K$ decays in contrast to $B \to KK$ [17] and $B \to KK^*$ [18] decays. The branching ratio in Scenario I is about twice as large as that in Scenario II for $B^0/\bar{B}^0 \to K_0^{*+}K^-$ while smaller than that in Scenario II for $B^0/\bar{B}^0 \to K^+K_0^{*-}$. As for the average of the two, the numerical prediction for $B^0 \to K_0^{*+}K^- + K^+K_0^{*-}$ in Scenario II is half of that in Scenario I.

(7) Except for $B^+ \to K^+\bar{K}_0^{*0}$ decay, where an upper limit is available now, there are no any experimental measurements for other decays considered here. We therefore

do not know which scenario is better now. The pQCD predictions for the branching ratios of $B \to K_0^* K$ decays will be tested by the LHC experiments.

In short, based on the assumption of two quark structure of scalar K_0^* meson, we calculated the branching ratios of $B \to K_0^* K$ decays at the leading order by using the pQCD factorization approach. From numerical calculations and phenomenological analysis, we found that the pQCD predictions for $Br(B^+ \to K^+ \bar{K}_0^{*0})$ is consistent with the existing experimental upper limit in both scenarios. We also predicted the branching ratios for other decay channels. All of these predictions will be tested by the LHC experiments. In the considered $B \to K_0^* K$ decays, the annihilation contributions played an important role, for $B^0 \to K_0^{*\pm} K^\mp$ modes, for example, which amount to $(1-4) \times 10^{-6}$.

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